

Localization of matter fields and non-Abelian gauge fields on domain walls

Masato ARAI^{a,b,*}) Filip BLASCHKE^{b,c,**}) Minoru ETO^{d,***}) and Norisuke SAKAI^{e,†})

^a*Fukushima National College of Technology, Iwaki, Fukushima 970-8034, Japan*

^b*Institute of Experimental and Applied Physics, Czech Technical University in Prague, Horská 22, 128 00 Prague 2, Czech Republic*

^c*Institute of Physics, Silesian University in Opava, Bezručovo nám. 1150/13, 746 01 Opava, Czech Republic*

^d*Department of Physics, Yamagata University, Yamagata 990-8560, Japan*

^e*Department of Mathematics, Tokyo Woman's Christian University, Tokyo 167-8585, Japan*

Abstract

We propose a method for simultaneous localization of non-Abelian gauge fields and matter fields with minimal interaction on domain walls using $U(N)_c$ gauge theory with $SU(N)_L \times SU(N)_R \times U(1)_A$ flavor symmetry. Localization of non-Abelian fields is achieved using field-dependent gauge coupling. We find that effective Lagrangian up to second order of derivatives for low energy fluctuations resembles a chiral model from hadron physics with additional minimal interactions of pions with localized gauge fields together with nonlinear interactions of moduli fields. This result provides a step towards a realistic model building of brane-world scenario using topological solitons.

^{*)} E-mail: masato.arai@gmail.com

^{**)} E-mail: filip.blaschke@fpf.slu.cz

^{***)} E-mail: meto@sci.kj.yamagata-u.ac.jp

^{†)} E-mail: norisuke.sakai@gmail.com

§1. Introduction

In the brane-world scenario^{1),2),3)} it is assumed that all Standard Model (SM) fields are localized on (3+1)-dimensional defect called 3-brane, which is embedded in multidimensional space-time (bulk), where only graviton may propagate freely. To realize this scenario dynamically, we may use a topological soliton. A domain wall is the simplest soliton in (4+1)-dimensional theory.

Generally, bulk fields can have massless and massive modes. It has been shown that massless modes localize naturally on the domain wall.⁴⁾ Integrating out all massive modes, one obtains effective field theory describing low energy interactions of these massless modes. However, in the context of field theory, localization of gauge fields, turned out to be difficult.⁵⁾ It has been noted that the broken gauge symmetry in the bulk outside of the soliton inevitably makes the localized gauge field massive with the mass of the order of inverse width of the wall.^{6),7)} Thus, to localize a massless gauge field, one needs to have the confining phase rather than the Higgs phase in the bulk outside of the soliton.

Recently, a classical picture of the confinement^{8),9)} realized by position-dependent gauge coupling has been used to localize the non-Abelian gauge field on domain walls.¹⁰⁾ This position-dependent gauge coupling was naturally introduced on the domain wall background through a scalar-field-dependent gauge coupling function resulting from a cubic prepotential of supersymmetric gauge theories. This mechanism in its simplest form uses two copies of $U(1)_c$ gauge theory, where the difference of respective kink-like profiles of adjoint scalars in both sectors is taken as position-dependent gauge coupling.

However, it is still an interesting problem to make localized matter fields and gauge fields interact. The purpose of this work is to present a (4+1)-dimensional field theory model of localized massless matter fields minimally coupled to the non-Abelian gauge field which is also localized on the domain wall with the (3+1)-dimensional world volume. We derive the low-energy effective field theory of these localized matter and gauge fields and describe full non-linear interaction between moduli fields.

To introduce non-Abelian flavor symmetry (to be gauged eventually) in the domain wall sector, we replace one of the two copies of the $U(1)_c$ gauge theory with the flavor symmetry $U(1)_L \times U(1)_R$ in,¹¹⁾ by $U(N)_c$ gauge theory with the extended flavor (global) symmetry $SU(N)_L \times SU(N)_R \times U(1)_A$. By choosing the coincident domain wall solution for this domain wall sector, we obtain the maximal unbroken non-Abelian flavor symmetry group $SU(N)_{L+R}$ which is preserved in both left and right vacua outside of the domain wall. Therefore we can introduce gauge field for the (subgroup of) the flavor $SU(N)_{L+R}$ symmetry. In order to obtain the field-dependent gauge coupling function, for the gauge field

localization mechanism,¹⁰⁾ we also introduce a coupling between a scalar field and gauge field strengths inspired by supersymmetric gauge theories, although we do not make the model fully supersymmetric at present. This scalar-field-dependent gauge coupling function gives appropriate profile of position-dependent gauge coupling through the background domain wall solution. With this localization mechanism for gauge field, we find massless non-Abelian gauge fields localized on the domain wall.

In what follows, we only concentrate on presenting our main result. Detailed discussion of involved calculations and other topic outside this main line is presented elsewhere.¹⁵⁾

§2. Gauge field localization

2.1. The domain wall sector

Let us briefly illustrate the localization mechanism for the gauge fields and the matter fields on the domain walls by using a simplest model in (4+1)-dimensional spacetime: two copies ($i = 1, 2$) of $U(1)$ models, each of which has two flavors (L, R) of charged Higgs scalar fields $H_i = (H_{iL}, H_{iR})$:

$$\mathcal{L}_i = -\frac{1}{4g_i^2} (\mathcal{F}_{MN}^i)^2 + \frac{1}{2g_i^2} (\partial_M \sigma_i)^2 + |\mathcal{D}_M H_i|^2 - V_i, \quad (2.1)$$

$$V_i = \frac{g_i^2}{2} (|H_i|^2 - v_i^2)^2 + |\sigma_i H_i - H_i M_i|^2. \quad (2.2)$$

We use the metric $\eta_{MN} = \text{diag}(+, -, \dots, -)$, $M, N = 0, 1, \dots, 4$. The Higgs field H_i is charged with respect to the $U(1)_i$ gauge symmetry and the covariant derivative is given by

$$\mathcal{D}_M H_i = \partial_M H_i + i w_M^i H_i, \quad (2.3)$$

where w_M^i is the $U(1)_i$ gauge field with the field strength

$$\mathcal{F}_{MN}^i = \partial_M w_N^i - \partial_N w_M^i. \quad (2.4)$$

Since we want domain walls, we will choose

$$M_i = \text{diag}(m_i, -m_i), \quad (m_i > 0), \quad (2.5)$$

resulting in the $U(1)_{iA}$ flavor symmetry. We have included the neutral scalar fields σ_i in this Abelian-Higgs model. Notice that the same coupling constant g_i appears not only in front of the kinetic terms of the gauge fields and σ_i , but also as the quartic coupling constant of H_i . We have taken this special relation among the coupling constants only to simplify

computations. One may repeat the whole procedure in models with more generic coupling constants without changing essential results.

There are two discrete vacua for each copy i

$$(H_{iL}, H_{iR}, \sigma_i) = (v_i, 0, m_i), \quad (0, v_i, -m_i). \quad (2.6)$$

Thanks to the special choice of the coupling constants in \mathcal{L}_i motivated by the supersymmetry, there are Bogomol'nyi-Prasad-Sommerfield (BPS) domain wall solutions in these models. Let y be the coordinate of the direction orthogonal to the domain wall and we assume all fields depend on only y . The BPS equations are

$$\partial_y \sigma_i + g_i^2 (|H_i|^2 - v_i^2) = 0, \quad \mathcal{D}_y H_i + \sigma_i H_i - H_i M_i = 0. \quad (2.7)$$

In order to obtain the domain wall solution interpolating the two vacua in Eq. (2.6), we impose the boundary conditions :

$$\begin{aligned} (H_{iL}, H_{iR}, \sigma_i) &= (0, v_i, -m_i), \quad y = -\infty, \\ (H_{iL}, H_{iR}, \sigma_i) &= (v_i, 0, m_i), \quad y = \infty. \end{aligned} \quad (2.8)$$

Tension T_i of the domain wall is given by a topological charge as

$$T_i = \int_{-\infty}^{\infty} dy \left[v_i^2 \partial_y \sigma_i - \partial_y \left((\sigma_i H_i - H_i M_i) H_i^\dagger \right) \right]_{-\infty}^{\infty} = 2m_i v_i^2. \quad (2.9)$$

The second equation of the BPS equations (2.7) can be solved by the moduli matrix formalism^{(12), (13)} with the constant matrix (vector) $H_{i0} = (C_{iL}, C_{iR})$

$$H_i = v_i e^{-\frac{\psi_i}{2}} H_{i0} e^{M_i y}, \quad \sigma_i + i w_i = \frac{1}{2} \partial_y \psi_i. \quad (2.10)$$

For a given H_{i0} , the scalar function ψ_i is determined by the master equation

$$\partial_y^2 \psi_i = 2g_i^2 v_i^2 \left(1 - e^{-\psi_i} H_{i0} e^{2M_i y} H_{i0}^\dagger \right). \quad (2.11)$$

There exists redundancy in the decomposition in Eq. (2.10), which is called the V -transformation:

$$H_{i0} \rightarrow V_i H_{i0}, \quad \psi_i \rightarrow \psi_i + 2 \log V_i, \quad V_i \in \mathbb{C}^*. \quad (2.12)$$

No analytic solutions for the master equation have been found for finite gauge couplings g_i . But we can take a strong gauge coupling limit $g_i \rightarrow \infty$ without losing any essential features (see⁽¹⁵⁾ for further details). The generic solutions of the domain wall are generated by the generic moduli matrices (after fixing the V -transformation)

$$H_{i0} = (C_{iL}, C_{iR}), \quad C_{iL}, C_{iR} \in \mathbb{C}^*. \quad (2.13)$$

The complex constants C_{iL}, C_{iR} are free parameters containing the moduli parameters of the BPS solutions. The moduli parameter can be defined by

$$C_i \equiv \sqrt{\frac{C_{iR}}{C_{iL}}} = e^{i\alpha_i} e^{m_i y_i}. \quad (2.14)$$

The other degrees of freedom in C_{iL}, C_{iR} can be eliminated by the V -transformation in Eq. (2.12) and has no physical meaning. Then, in the strong gauge coupling limit, the master equation is found to be (where we used coincident walls set-up $H_{i0} = (1, 1)$)

$$1 = e^{-\psi_i} (e^{2m_i(y-y_i)} + e^{-2m_i(y-y_i)}) , \quad (2.15)$$

with the solution

$$\sigma_i = m_i \frac{1 - |\phi_i|^2}{1 + |\phi_i|^2} = m_i \tanh 2m_i(y - y_i). \quad (2.16)$$

It is obvious that the real parameter y_i is the translational moduli of the domain wall. The other parameter α_i is an internal moduli which is the Nambu-Goldstone (NG) mode associated with the $U(1)_{iA}$ flavor symmetry spontaneously broken by the domain walls.

Let us next derive the low energy effective theory on the domain wall. We integrate all the massive modes while keeping the massless modes. We use the so-called moduli approximation where the dependence on (3+1)-dimensional spacetime coordinates comes into the effective Lagrangian only through the moduli fields:

$$C_i \rightarrow C_i(x^\mu), \quad \phi_i(y) \rightarrow \phi_i(y, C_i(x^\mu)) = C_i(x^\mu)^2 e^{-2m_i y}. \quad (2.17)$$

The effective Lagrangian for the moduli field $C_i(x^\mu) = e^{i\alpha_i} e^{m_i y_i}$ is given by

$$\mathcal{L}_{i,\text{eff}} = \frac{2m_i v_i^2}{2} (\partial_\mu y_i)^2 + \frac{v_i^2}{m_i} (\partial_\mu \alpha_i)^2, \quad (2.18)$$

where energy of soliton solution is neglected since it does not contribute to dynamics of moduli. Note that $2m_i v_i^2$ is precisely the domain wall tension. This is the free field Lagrangian.

In order to obtain the massless gauge field to be localized on the domain wall, we need a new gauge symmetry which is unbroken in the bulk. Recently, a new mechanism was proposed to localize gauge fields on domain walls.¹⁰⁾

A key ingredient is the so-called dielectric coupling constant.^{8),9)} Let us introduce a new $U(1)$ gauge field a_M which we wish to localize on the domain wall. Since this gauge symmetry should be unbroken in the bulk, we consider the case where all the Higgs fields are neutral under this newly introduced $U(1)$ gauge symmetry. The gauge field a_M is assumed to couple to the neutral scalar fields σ_i only in the following particular combination

$$\mathcal{L} = \mathcal{L}_1 + \mathcal{L}_2 - \frac{\lambda}{2} \left(\frac{\sigma_1}{m_1} - \frac{\sigma_2}{m_2} \right) (\mathcal{G}_{MN})^2, \quad (2.19)$$

where a real constant λ with the unit mass dimension, in accordance with the (4+1)-dimensional spacetime and the field strength is defined by

$$\mathcal{G}_{MN} = \partial_M a_N - \partial_N a_M. \quad (2.20)$$

The field-dependent gauge coupling function is given by

$$\frac{1}{4e^2(\sigma)} = \frac{\lambda}{2} \left(\frac{\sigma_1}{m_1} - \frac{\sigma_2}{m_2} \right), \quad (2.21)$$

which depends on the position y through fields σ_i . Thus the field-dependent gauge coupling function $e(\sigma)$ plays the role of the dielectric coupling constant. The form of Eq. (2.21) is chosen so that the gauge interaction becomes strongly coupled in the bulk ($\sigma_i \rightarrow \pm m_i$ as $y \rightarrow \pm\infty$).

Since our Lagrangian has no term linear in a_M , the equations of motion for a_M is trivially solved by $a_M = 0$, and the rest of the equations of motion are explicitly the same as those in the previous subsection. Therefore the domain wall solution in the previous subsection together with $a_M = 0$ is still a solution of the equations of motion. Thus, the (3+1)-dimensional gauge coupling constant is given by

$$\frac{1}{4e_4^2} = \frac{\lambda}{2} \int_{-\infty}^{\infty} dy \left(\frac{\sigma_1}{m_1} - \frac{\sigma_2}{m_2} \right) = \frac{\lambda}{4} \left[\frac{\psi_1}{m_1} - \frac{\psi_2}{m_2} \right]_{-\infty}^{\infty} = \lambda(y_2 - y_1), \quad (2.22)$$

where we have used the asymptotic behavior $\psi_i \rightarrow \log 2 \cosh 2m_i(y - y_i)$ as $|y| \rightarrow \infty$. The low energy effective Lagrangian is found to be

$$\mathcal{L}_{\text{eff}} = \sum_{i=1,2} \left[\frac{2m_i v^2}{2} (\partial_\mu y_i)^2 + \frac{v_i^2}{m_i} (\partial_\mu \alpha_i)^2 \right] - \lambda(y_2 - y_1) (\mathcal{G}_{\mu\nu})^2. \quad (2.23)$$

Although we succeeded in localizing the massless $U(1)$ gauge field a_μ on the domain walls, the Lagrangian Eq. (2.23) has no charged matter fields minimally coupled with the localized gauge field a_μ . In the next section, we will give a model with a non-Abelian global symmetry whose unbroken subgroup can be gauged to yield massless localized gauge fields on the domain wall.

§3. The chiral model

In this section we study domain walls in the chiral model which is a natural extension of the Abelian-Higgs model in the previous section.

Let us consider the Yang-Mills-Higgs model with $SU(N)_c \times U(1)$ gauge symmetry with $S[U(N)_L \times U(N)_R] = SU(N)_L \times SU(N)_R \times U(1)_A$ flavor symmetry.^{14),11)} To localize the

gauge field in a simple manner, we again introduce two sectors \mathcal{L}_1 and \mathcal{L}_2 , but only the former is extended to Yang-Mills-Higgs system and the latter is the same form as in (2.1) with $i = 2$. The matter contents are summarized in Table I. Since the presence of two factors of $SU(N)$ global symmetry resembles the chiral symmetry of QCD, we call this Yang-Mills-Higgs system as the chiral model.

	$SU(N)_c$	$U(1)_1$	$U(1)_2$	$SU(N)_L$	$SU(N)_R$	$U(1)_{1A}$	$U(1)_{2A}$	mass
H_{1L}	\square	1	0	\square	$\mathbf{1}$	1	0	$m_1 \mathbf{1}_N$
H_{1R}	\square	1	0	$\mathbf{1}$	\square	-1	0	$-m_1 \mathbf{1}_N$
Σ_1	$\text{adj} \oplus \mathbf{1}$	0	0	$\mathbf{1}$	$\mathbf{1}$	0	0	0
H_{2L}	$\mathbf{1}$	0	1	$\mathbf{1}$	$\mathbf{1}$	0	1	m_2
H_{2R}	$\mathbf{1}$	0	1	$\mathbf{1}$	$\mathbf{1}$	0	-1	$-m_2$
Σ_2	$\mathbf{1}$	0	0	$\mathbf{1}$	$\mathbf{1}$	0	0	0

Table I. Quantum numbers of the domain wall sectors in the chiral model.

The Lagrangian is given by

$$\mathcal{L} = \mathcal{L}_1 + \mathcal{L}_2, \quad (3.1)$$

$$\mathcal{L}_1 = \text{Tr} \left[-\frac{1}{2g_1^2} (F_{1MN})^2 + \frac{1}{g_1^2} (\mathcal{D}_M \Sigma_1)^2 + |\mathcal{D}_M H_1|^2 \right] - V_1, \quad (3.2)$$

$$V_1 = \text{Tr} \left[\frac{g_1^2}{4} \left(H_1 H_1^\dagger - v_1^2 \mathbf{1}_N \right)^2 + |\Sigma_1 H_1 - H_1 M_1|^2 \right], \quad (3.3)$$

with $H_1 = (H_{1L}, H_{2L})$. \mathcal{L}_2 is the same form as (2.1) with $i = 2$. Gauge fields of $U(N)_c = (SU(N)_c \times U(1)_1)/Z_N$ are denoted as W_{1M} , and adjoint scalar as Σ_1 . The covariant derivative and the field strength are denoted as $\mathcal{D}_M \Sigma_1 = \partial_M \Sigma_1 + i[W_{1M}, \Sigma_1]$, $\mathcal{D}_M H_1 = \partial_M H_1 + iW_{1M} H_1$, and $F_{1MN} = \partial_M W_{1N} - \partial_N W_{1M} + i[W_{1M}, W_{1N}]$. The mass matrix is given by $M_1 = \text{diag}(m_1 \mathbf{1}_N, -m_1 \mathbf{1}_N)$. Let us note that the chiral model reduces to the Abelian-Higgs model in the limit of $N \rightarrow 1$, by deleting all the $SU(N)$ groups.

The role of the second sector is to provide the localization mechanism in the similar way as in the previous section. Therefore, the remaining discussion will be focused solely on the first sector ($i = 1$) and we suppress the index $i = 1$.

There exist $N + 1$ vacua in which the fields develop the following VEV

$$H = (H_L, H_R) = v \left(\begin{array}{c|c} \mathbf{1}_{N-r} & \mathbf{0}_{N-r} \\ \hline \mathbf{0}_r & \mathbf{1}_r \end{array} \right), \quad (3.4)$$

$$\Sigma = m \left(\begin{array}{c} \mathbf{1}_{N-r} \\ -\mathbf{1}_r \end{array} \right), \quad (3.5)$$

with $r = 0, 1, 2, \dots, N$. We refer these vacua with the label r . In the r -th vacuum, both the local gauge symmetry $U(N)_c$ and the global symmetry are broken, but a diagonal global symmetries are unbroken (color-flavor-locking)

$$\begin{aligned} U(N)_c \times SU(N)_L \times SU(N)_R \times U(1)_A \rightarrow \\ SU(N-r)_{L+c} \times SU(r)_L \times SU(r)_{R+c} \times SU(N-r)_R \times U(1)_{A+c}. \end{aligned} \quad (3.6)$$

The BPS equations for the domain walls can be obtained as:

$$\mathcal{D}_y \Sigma - \frac{g^2}{2} (v^2 \mathbf{1}_N - H H^\dagger) = 0, \quad (3.7)$$

$$\mathcal{D}_y H + \Sigma H - H M = 0. \quad (3.8)$$

The tension of the domain wall is given by

$$\begin{aligned} T &= \int_{-\infty}^{\infty} dy \partial_y \{ \text{Tr} [v^2 \Sigma - (\Sigma H - H M) H^\dagger] \} \\ &= v^2 \text{Tr} [\Sigma(+\infty) - \Sigma(-\infty)]. \end{aligned} \quad (3.9)$$

Let us concentrate on the domain wall which connects the 0-th vacuum at $y \rightarrow \infty$ and the N -th vacuum at $y \rightarrow -\infty$. Its tension can be read as

$$T = 2Nv^2m, \quad (3.10)$$

from Eq. (3.9). Since there are $N + 1$ possible vacua, the maximal number of walls is N at various positions. The simplest domain wall solution corresponding to the coincident walls is given by making an ansatz that H_L , H_R , Σ and W_y are all proportional to the unit matrix. Then the BPS equations (3.7) and (3.8) can be identified with the BPS equations in Eq. (2.7) in the Abelian-Higgs model. Thus the domain wall solution can be solved as

$$H_L = v e^{-\frac{\psi}{2}} e^{my} \mathbf{1}_N, \quad (3.11)$$

$$H_R = v e^{-\frac{\psi}{2}} e^{-my} \mathbf{1}_N, \quad (3.12)$$

$$\Sigma + iW_y = \frac{1}{2} \partial_y \psi \mathbf{1}_N, \quad (3.13)$$

where ψ is the solution of the master equation (2.11) in the Abelian-Higgs model. Eq.(3.6) shows that the unbroken global symmetry for N -th vacuum ($H_L = 0$, $H_R = v \mathbf{1}_N$ and $\Sigma = -m \mathbf{1}_N$) at the left infinity $y \rightarrow -\infty$ is $SU(N)_L \times SU(N)_{R+c} \times U(1)_{A+c}$, whereas that for the 0-th vacuum ($H_L = v \mathbf{1}_N$, $H_R = 0$ and $\Sigma = m \mathbf{1}_N$) at the right infinity $y \rightarrow \infty$ is $SU(N)_{L+c} \times SU(N)_R \times U(1)_{A+c}$.

The domain wall solution further breaks these unbroken symmetries because it interpolates the two vacua.

$$U(N)_c \times SU(N)_L \times SU(N)_R \times U(1)_A \rightarrow SU(N)_{L+R+c}. \quad (3.14)$$

This spontaneous breaking of the global symmetry gives NG modes on the domain wall as massless degrees of freedom valued on the coset similarly to the chiral symmetry breaking in QCD :

$$\frac{SU(N)_L \times SU(N)_R}{SU(N)_{L+R+c}} \times U(1)_A. \quad (3.15)$$

Since our model can be embedded into a supersymmetric field theory,¹⁵⁾ these NG modes ($U(N)$ chiral fields) appear as complex scalar fields accompanied with additional N^2 pseudo-NG modes*).

We will now give the low-energy effective Lagrangian on the domain walls where the massless moduli fields (the matter fields) are localized. The best way to parametrize these massless moduli fields is to use the moduli matrix formalism^{12), 13)}

$$H_L = v e^{my} S^{-1}, \quad (3.16)$$

$$H_R = v e^{-my} S^{-1} e^\phi, \quad (3.17)$$

$$\Sigma + iW_y = S^{-1} \partial_y S, \quad (3.18)$$

where $S \in GL(N, \mathbf{C})$ and $\Omega = SS^\dagger$ is the solution of the following master equation

$$\partial_y (\Omega^{-1} \partial_y \Omega) = g^2 v^2 (\mathbf{1}_N - \Omega^{-1} \Omega_0), \quad (3.19)$$

where

$$\Omega_0 = e^{2my} \mathbf{1}_N + e^{-2my} e^\phi e^{\phi^\dagger}.$$

We have used the V -transformation to identify the moduli e^ϕ , which is a complex N by N matrix. It can be parametrized by an $N \times N$ hermitian matrix \hat{x} and a unitary matrix U as¹¹⁾

$$e^\phi = e^{\hat{x}} U^\dagger, \quad (3.20)$$

where U is nothing but the $U(N)$ chiral fields associated with the spontaneous symmetry breaking Eq. (3.15) and \hat{x} is the pseudo-NG modes whose existence we promised above.

*) One of them is actually a genuine NG mode corresponding to the broken translation.

In the strong gauge coupling limit $g \rightarrow \infty$, a solution of master equation is simply $\Omega = \Omega_0$. After fixing the $U(N)_c$ gauge, we obtain

$$S = e^{\hat{x}/2} \sqrt{2 \cosh(2my - \hat{x})}. \quad (3.21)$$

Let us denote, for brevity

$$\hat{y} = 2my - \hat{x}, \quad (3.22)$$

the Higgs fields are then given as

$$H_L = v \frac{e^{\hat{y}/2}}{\sqrt{2 \cosh \hat{y}}}, \quad (3.23)$$

$$H_R = v \frac{e^{-\hat{y}/2}}{\sqrt{2 \cosh \hat{y}}} U^\dagger. \quad (3.24)$$

From this solution, one can easily recognize that eigenvalues of \hat{x} correspond to the positions of the N domain walls in the y direction. Now we promote moduli parameters \hat{x} and U to fields on the domain wall world volume, namely functions of world volume coordinates x^μ . We plug the domain wall solutions $H_{L,R}(y; \hat{x}(x^\mu), U(x^\mu))$ into the original Lagrangian \mathcal{L} in Eq. (3.2) at $g \rightarrow \infty$ and pick up the terms quadratic in the derivatives. Thus the low energy effective Lagrangian is given by

$$\mathcal{L}_{\text{eff}} = \int_{-\infty}^{\infty} dy \text{Tr} \left[\partial_\mu H_L \partial^\mu H_L^\dagger + \partial_\mu H_R \partial^\mu H_R^\dagger - v^2 W_\mu W^\mu \right], \quad (3.25)$$

where

$$W_\mu = \frac{i}{2v^2} \left[\partial_\mu H_L H_L^\dagger - H_L \partial_\mu H_L^\dagger + (L \leftrightarrow R) \right]. \quad (3.26)$$

Here we have eliminated the massive gauge field W_μ by using the equation of motion. Let us next introduce the gauge fields which are to be massless and localized on the domain walls. Since associated gauge symmetry should not be broken by the domain walls, we can gauge only unbroken symmetry $SU(N)_{L+R+c}$ itself or its subgroup. Let us gauge $SU(N)_{L+R} \equiv SU(N)_V$ and let A_μ^a be the $SU(N)_{L+R}$ gauge field. The Higgs fields are in the bi-fundamental representation of $U(N)_c$ and $SU(N)_{L+R}$. The covariant derivatives of the Higgs fields are modified by

$$\tilde{\mathcal{D}}_M H_{1L} = \partial_M H_{1L} + iW_{1M} H_{1L} - iH_{1L} A_M, \quad (3.27)$$

$$\tilde{\mathcal{D}}_M H_{1R} = \partial_M H_{1R} + iW_{1M} H_{1R} - iH_{1R} A_M. \quad (3.28)$$

We now introduce a field-dependent gauge coupling function $g^2(\Sigma)$ for A_M , which is inspired by the supersymmetric model in Ref.¹⁰⁾

$$\frac{1}{2e^2(\Sigma)} = \frac{\lambda}{2} \left(\frac{\text{Tr} \Sigma_1}{Nm_1} - \frac{\Sigma_2}{m_2} \right). \quad (3.29)$$

The Lagrangian is given by

$$\mathcal{L} = \tilde{\mathcal{L}}_1 + \mathcal{L}_2 - \frac{1}{2e^2(\Sigma)} \text{Tr} [G_{MN} G^{MN}]. \quad (3.30)$$

The $\tilde{\mathcal{L}}_1$ in Eq. (3.30) is given by Eq. (3.2) where the covariant derivatives are replaced with those in Eqs. (3.27) and (3.28).

We first wish to find the domain wall solutions in this extended model. As before, we make ansatz that all the fields depend on only y and $W_\mu = A_\mu = 0$. Let us first look on the E.O.M. of the new gauge field A_M . It is of the form

$$\mathcal{D}_M G^{MN} = J^N, \quad (3.31)$$

where J_M stands for the current of A_M . Note that the current J_M is zero, by definition, if we plug the domain wall solutions in the chiral model before gauging the $SU(N)_{L+R}$. This is because the domain wall configurations do not break $SU(N)_{L+R}$. Therefore, $A_M = 0$ is a solution of Eq. (3.31).

Then, we are left with equation of motion with $A_M = 0$ which are identical to those in the ungauged chiral model in the previous subsections. Therefore the gauged chiral model admits the same domain wall solutions as those (Eqs. (3.23) and (3.24)) in the ungauged chiral model.

The next step is to derive the low energy effective theory on the domain wall world-volume in the moduli approximation as before. We just repeat the similar computation presented above. It is convenient to define the covariant derivative only for the flavor ($SU(N)_{L+R}$) gauge interactions as

$$\hat{D}_\mu H = \partial_\mu H - iH A_\mu. \quad (3.32)$$

Then we obtain the effective Lagrangian of the first sector as

$$\begin{aligned} \mathcal{L}_{1,\text{eff}} = & \int_{-\infty}^{\infty} dy \text{Tr} \left[\hat{D}_\mu H_L (\hat{D}^\mu H_L)^\dagger + \hat{D}_\mu H_R (\hat{D}^\mu H_R)^\dagger - v^2 W_\mu W^\mu \right. \\ & \left. - \frac{1}{2e^2(\Sigma)} G_{MN} G^{MN} \right], \end{aligned} \quad (3.33)$$

with

$$W_\mu = \frac{i}{2v^2} \left[\hat{D}_\mu H_L H_L^\dagger - H_L (\hat{D}_\mu H_L)^\dagger + (L \leftrightarrow R) \right]. \quad (3.34)$$

Eliminating W_μ , we obtain the following expression for the integrand of the effective Lagrangian after some simplification

$$\mathcal{L}_{\text{eff}} = \frac{1}{2v^2} \int_{-\infty}^{\infty} dy \text{Tr} \left[\mathcal{D}_\mu H_{ab} \mathcal{D}^\mu H_{ba} \right], \quad (3.35)$$

where we defined fields H_{ab} with the label ab of adjoint representation of the flavor gauge group $SU(N)_{L+R+c}$ and the covariant derivative as

$$\mathcal{D}_\mu H_{ab} = \partial_\mu H_{ab} + i[A_\mu, H_{ab}], \quad H_{ab} \equiv H_a^\dagger H_b, \quad a, b = L, R. \quad (3.36)$$

The full description of the procedure to obtain the effective Lagrangian in the closed form can be found in.¹⁵⁾ Here we merely state the result:

$$\begin{aligned} \mathcal{L}_{1,\text{eff}} = & \frac{v^2}{2m} \text{Tr} \left[\mathcal{D}_\mu \hat{x} \frac{\cosh(\mathcal{L}_{\hat{x}}) - 1}{\mathcal{L}_{\hat{x}}^2 \sinh(\mathcal{L}_{\hat{x}})} \ln \left(\frac{1 + \tanh(\mathcal{L}_{\hat{x}})}{1 - \tanh(\mathcal{L}_{\hat{x}})} \right) (\mathcal{D}^\mu \hat{x}) \right. \\ & + U^\dagger \mathcal{D}_\mu U \frac{\cosh(\mathcal{L}_{\hat{x}}) - 1}{\mathcal{L}_{\hat{x}} \sinh(\mathcal{L}_{\hat{x}})} \ln \left(\frac{1 + \tanh(\mathcal{L}_{\hat{x}})}{1 - \tanh(\mathcal{L}_{\hat{x}})} \right) (\mathcal{D}^\mu \hat{x}) \\ & \left. + \frac{1}{2} \mathcal{D}_\mu U^\dagger U \frac{1}{\tanh(\mathcal{L}_{\hat{x}})} \ln \left(\frac{1 + \tanh(\mathcal{L}_{\hat{x}})}{1 - \tanh(\mathcal{L}_{\hat{x}})} \right) (U^\dagger \mathcal{D}^\mu U) \right], \end{aligned} \quad (3.37)$$

where

$$\mathcal{L}_A(B) = [A, B] \quad (3.38)$$

is a Lie derivative with respect to A . The covariant derivative \mathcal{D}_μ is defined by

$$\mathcal{D}_\mu U = \partial_\mu U + i[A_\mu, U]. \quad (3.39)$$

By expanding (3.37), we here illustrate nonlinear interactions of \hat{x} up to fourth orders in the fluctuations \hat{x} and $U - 1$

$$\begin{aligned} \mathcal{L}_{1,\text{eff}} = & \frac{v^2}{2m} \text{Tr} \left(\mathcal{D}_\mu U^\dagger \mathcal{D}^\mu U + \mathcal{D}_\mu \hat{x} \mathcal{D}^\mu \hat{x} + U^\dagger \mathcal{D}_\mu U [\hat{x}, \mathcal{D}^\mu \hat{x}] \right. \\ & \left. - \frac{1}{12} [\mathcal{D}_\mu \hat{x}, \hat{x}] [\hat{x}, \mathcal{D}^\mu \hat{x}] + \frac{1}{3} [\mathcal{D}_\mu U^\dagger U, \hat{x}] [\hat{x}, U^\dagger \mathcal{D}^\mu U] + \dots \right). \end{aligned} \quad (3.40)$$

In a similar way to Eq.(2.22), we can define the (3+1)-dimensional non-Abelian gauge coupling e_4 by integrating (3.29) and find

$$\frac{1}{2e_4^2} = \int dy \frac{1}{2e^2(\Sigma)} = \lambda(y_2 - y_1), \quad (3.41)$$

where y_i is the wall position for the i -th domain wall sector. Summarizing, we obtain the following effective Lagrangian

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{1,\text{eff}} + \mathcal{L}_{2,\text{eff}} - \frac{1}{2e_4^2} \text{Tr} [G_{\mu\nu} G^{\mu\nu}], \quad (3.42)$$

where $\mathcal{L}_{2,\text{eff}}$ is given in (2.18). This is the main result. We have succeeded in constructing the low energy effective theory in which the matter fields (the chiral fields) and the non-Abelian gauge fields are localized with the non-trivial interaction. We show the profile of

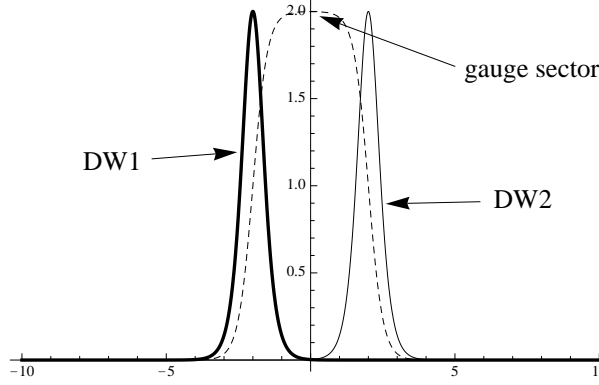


Fig. 1. The wave functions of the zero modes. DW1 and DW2 stand for the wave functions of the massless matter fields of the $i = 1$ domain wall and $i = 2$ domain wall, respectively for strong gauge coupling limit $g_i = \infty$ and $m_i = 1$. The gauge fields are localized between the domain walls.

”wave functions” of localized massless gauge field and massless matter fields as functions of the coordinate y of the extra dimension in Fig. 1.

As is seen from Eq.(3.40), the flavor gauge symmetry $SU(N)_{L+R+c}$ is further (partly) broken and the corresponding gauge field A_μ becomes massive, when the fluctuation $\phi = e^{\hat{x}}U$ develops non-zero vacuum expectation values. Especially, \hat{x} is interesting because its non-vanishing (diagonal) values of the fluctuation has the physical meaning as the separation between walls away from the coincident case. For instance, if all the walls are separated, $SU(N)_{L+R+c}$ is spontaneously broken to the maximal $U(1)$ subgroup $U(1)^{N-1}$. However, if r walls are still coincident and all other walls are separated, we have an unbroken gauge symmetry $SU(r) \times U(1)^{N-r+1}$. Then, a part of the pseudo-NG modes \hat{x} turn to NG modes associated with the further symmetry breaking $SU(N)_{L+R+c} \rightarrow SU(r) \times U(1)^{N-r+1}$, so that the total number of zero modes^{*)} is preserved.¹¹⁾ These new NG modes, called the non-Abelian cloud, spread between the separated domain walls.¹¹⁾ The flavor gauge fields eat the non-Abelian cloud and get masses which are proportional to the separation of the domain walls. This is the Higgs mechanism in our model. This geometrical understanding of the Higgs mechanism is quite similar to D-brane systems in superstring theory. So our domain wall system provides a genuine prototype of field theoretical D3-branes.

^{*)} In Ref.¹⁴⁾ the authors argued that the non-Abelian clouds spreading between walls become massive contrary to the results of.¹¹⁾

§4. Conclusions and discussion

In this work we have successfully localized both massless non-Abelian gauge fields and massless matter fields in non-trivial representation of the gauge group. Then we studied the low-energy effective Lagrangian and showed that massless localized matter fields interact minimally with localized $SU(N)_{L+R}$ gauge field as adjoint representations. Moreover, full nonlinear interaction between the moduli containing up to the second derivatives, was worked out.

Main result of this paper is the effective Lagrangian (3.42). The moduli field U appearing in the effective theory, is a chiral $N \times N$ matrix field like a pion, since it is a NG boson of spontaneously broken chiral symmetry. Other moduli in (3.42), denoted by $N \times N$ Hermitian matrix \hat{x} , has the physical meaning of positions of N domain walls as its diagonal elements. We argued that the fluctuations of moduli field \hat{x} , can develop VEV corresponding to splitting of walls, and the Higgs mechanism will occur as a result. Namely, the flavor gauge fields get masses by eating the non-Abelian cloud. Therefore, in this model, Higgs mechanism has a geometrical origin like low energy effective theories on D-branes in superstring theory.

Amongst the possible future investigations, we would like to study non-coincident solution to further clarify this geometrical Higgs mechanism.

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